Name—————————————, **ID** ——————

HW Number ONE, DUE: Sunday Feb 29, 2012 at 2pm

Ayman Badawi

QUESTION 1. a) Construct the Caley's Table for the group (D_3, o).

b) Find the order of each element in D_3

c) Is it true that $|a\ o\ b| = |a||b|$ for every $a, b \in D_3$? EXPLAIN

d) Let $F = \{a \in D_5 \mid a$ is a rotation$\}$. For each $b \in F$ find $|b|$.

e) Let $F = \{a \in D_8 \mid a$ is a rotation $\}$. For each $b \in F$ find $|b|$ (something is going on and it seems what you might concluded from (d) not always TRUE!!!)

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HW Number TWO, DUE: Sunday March 4, 2012 at 2pm

Ayman Badawi

QUESTION 1. Let $S = \{2, 4, 8, 10, 14, 16\} \subset Z_{18}^*$. constructing the Caley table for (S, X_{18}). After you do that it will be clear that that (S, X_{18}) is a group. Now

a) Find the identity of S.

b) For each $b \in S$, find b^{-1}.

c) Is S cyclic? If yes write $S =< a >$ for some $a \in S$

QUESTION 2. Let S be a nonempty finite subset of a group $(G, *)$. Suppose that $(S, *)$ is closed. Prove that $(S, *)$ is a subgroup of $(G, *)$[Hint: you only need to show that $e \in S$. Then by a result in the class, we are done].

QUESTION 3. Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} \in M_2(Z_7)$. Find A^{-1}.

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HW Number Three, DUE: Sunday March 18, 2012 at 2pm

Ayman Badawi

QUESTION 1. a) Let $(G, *)$ be a finite cyclic group with m elements ($m < \infty$). Let d be a factor of m (i.e., $d \mid m$). Show that G has exactly one subgroup of order d.

b) Suppose that $(G, *)$ is an infinite cyclic group. Prove that G has exactly two generators.

c) Find $|10|, |13|, |103|, |25|$ in $(Z_{225}, +)$.

d) Let G be a group and $a \in G$ such that $|a| = m < \infty$ and let $1 \le k < m$. Prove that $|a^k| = |a^{m-k}|$.

e) Let $a, b \in (G, *)$ such that $|a| = m < \infty$. Prove that $|a| = |b * a * b^{-1}| = m$.

f) In $D_3, |rot_{120}| = 3, |ref| = 2, gcd(2, 3) = 1$ but $|rot_{120} \ \ 0 \ \ ref| \neq |rot_{120}||ref|$. Now let $(G, *)$ be a group, $a, b \in G$ such that $a * b = b * a$, $|a| = m, |b| = n$ and $gcd(m, n) = 1$. Prove that $|a * b| = |a||b| = mn$.

g) Let $S = \{\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \mid k \in Z_{18}\}$. Prove that (S, X) is a cyclic group with exactly 18 elements (here X means matrix multiplication). Find all generators of S.

h) Is $(U(10), \times)$ cyclic? if yes, find all generators of $U(10)$. Is $U(9)$ cyclic? find all generators of $U(9)$.

i) Let $n \ge 3$. Find $|2^{n-1} - 1|, |2^{n-1} + 1|$ in $(U(2^n), \times)$. Now prove that $(U(2^n), \times)$ is not cyclic. [hint : use (a) above].

f+) Let G be a finite abelian group with 36 elements. Given $a, b \in G$ such that $|a| = 9$ and $|b| = 4$. Prove that G is cyclic

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HW Number Four, DUE: Thursday March 29, 2012 at 2pm

Ayman Badawi

QUESTION 1. Let $G = (U(25), \times)$. Calculate:
a) $|2|$
b) $|16|$
c) $|7|$
d) Let H be a subgroup of G with exactly 4 elements. Calculate all distinct left cosets of H.

QUESTION 2. Construct an infinite non-abelian group G such that for each positive integer $n \geq 2$, G has a cyclic finite subgroup of order n.

Here are the steps to construct such example:

a) Let $M_2(R)$ be the set of all 2×2 matrices with entries from R (real numbers). Let $G = \{A \in M_2(R) \mid det(A) = 1\}$. Show that G under normal matrix multiplication is a non-abelian group. (i.e, Show that G is closed, no need to show associative (it is always associative), show that G has identity, show that for each $A \in G$, A^{-1} exist and $A^{-1} \in G$.). It should be clear that G is non-abelian. Just find two elements A, B in G and check that $AB \neq BA$.

b) Let $n \geq 2$. Let $F = \begin{bmatrix} cos(360/n) & -sin(360/n) \\ sin(360/n) & cos(360/n) \end{bmatrix}$. Show that $|F| = n$, and hence $< F >$ is a finite cyclic subgroup of G with exactly n elements. [Note that $cos(a)cos(b) - sin(a)sin(b) = cos(a + b)$ and $sin(a)cos(b) + cos(a)sin(b) = sin(a + b)$]

c) In view of (b), let $D = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$. Find $|D|$.

QUESTION 3. Let H, D be finite groups and $M = H \bigoplus D$.
a) prove that $|(a, b)| = LCM(|a|, |b|)$ [NICE TO HAVE!!!]
b) Now let $M = U(8) \bigoplus Z_3$. Find $|(3, 2)|$. Construct a subgroup of M with exactly 4 elements, say H. Then find all distinct left cosets of H.

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HW number 5, MTH 320: Abstract Algebra I, Spring 2012, Due: Sunday May 3^{rd} at 2pm

Ayman Badawi

QUESTION 1. Which of the following groups is (are) cyclic. If cyclic, then find a generator. If not cyclic, then briefly give me a reason.

(i) Let $G = Z_4 \oplus U(27)$

(ii) Let $G = D_3 \oplus Z_7$

(iii) Let $G = A_3 \oplus Z_8$

(iv) Let $G = U(4) \oplus A_3$

QUESTION 2. 1) Show that A_4 has exactly one subgroup of order 4. Construct such group and call it H. Show that H is a normal subgroup of A_4 and Find the elements of A_4/H. **IMPORTANT: (You should know that a group G is simple if the only proper normal subgroup of G is $\{e\}$. So the correct statement about A_n: A_3 is simple and A_n is simple for each $n >= 5$. The only exception is A_4).**

QUESTION 3. It is easy to see that $M = GL(2, Z)$ is a group and $H = SL(2, Z)$ is a normal subgroup of $M = GL(2, Z)$. Find all elements of G/H and show that G/H is cyclic (and hence abelian).

QUESTION 4. It is easy to see that (Z, +) is a normal subgroup of $(Q, +)$ and hence G/Z is a group. Show that G/Z is infinite such that each element in G/Z has a finite order. In particular, for each positive integer $n \geq 2$, show that G/Z has exactly $\phi(n)$ elements where each element has order n.

QUESTION 5. (EXTRA CREDIT = 5 points) Show that S_4 does not contain a normal subgroup of order 8

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